1.3.1 Geometric Sequences Homework 1 Name:

Practice Set A – Routine Practice

1. The first term of a geometric sequence is 7 and the common ratio is 4.
   Find the 6th term.
2. In a geometric sequence, $u_1 = 1250,$ and $r = 0.8.$ Find the 5th term.
3. A geometric sequence has $u_3 = 81,$ and $r = 3.$ Find the value of u_1.
4. Find the sum of the first 5 terms of the geometric series where the first term is 2 and $r = 5$.
5. A geometric series has $u_1 = 640, r = \frac{1}{2}$. Find the sum of the first 6 terms. Write your answer in standard form.
6. The 4th term of a geometric series is 54 and the common ratio is 3.
   Find the sum of the first 4 terms.

Practice Set B – IB-Style Questions (10 Total)

1. Leo starts a lawn mowing business during the summer. On his first day, he charges \$40 per lawn and completes 2 lawns. Each day, he increases the number of lawns by 1 and raises his rate by \$5 per lawn.

   By the 6th day, Leo is charging \$65 per lawn.

   a. (i) Show that Leo charges \$45 per lawn on the second day.
      (ii) Find the expression for the amount he charges per lawn on day n.
      (iii) Find the number of lawns he mows on day n. [5]

   b. On which day will Leo first charge more than \$100 per lawn? [2]
   c. On which day will Leo first mow 10 lawns in a single day? [2]

2. Ava subscribes to a digital art platform. The subscription starts at \$10 per month. Each year, the subscription cost increases by \$2 per month. She keeps the subscription for 4 years.

   a. Show that Ava pays \$10 per month during the first year and \$12 per month during the second year.
      (ii) Write an expression for the monthly cost during year n.
      (iii) Hence, calculate the total cost Ava pays over the 4 years. [5]

b. After 4 years, Ava cancels her subscription. Her friend Bea signs up at the same starting rate of $10 per month but her plan increases by 5% each year.
    i. Write an expression for Bea's monthly cost during year n. [2]
    ii. Calculate Bea's monthly cost in the third year, rounded to the nearest penny. [2]
    iii. Compare Ava's and Bea's monthly costs during the fourth year. Who is paying more? [2]

3. A group of friends is training for a cycling race. On the first day, they ride 12 km. Each day, they increase their distance by a **constant factor** of 1.15.
    a. Write an expression for the distance ridden on day n. [1]
    b. On which day will they first ride more than **50 km**? [2]
    c. Determine the distance ridden on day 7, rounded to one decimal place. [2]

4. A coffee shop offers a loyalty program where the price per coffee starts at $3.20 and increases by **10%** each year due to inflation.
    a. Write an expression for the price of a coffee during year n. [1]
    b. Find the price of a coffee in the **5th year**, to the nearest cent. [2]
    c. In which year will the price first exceed $5? [2]

5. A scientist observes a bacterial culture. The initial population is 1,200 cells, and the population triples every 8 hours.
    a. Write an expression for the population after n 8-hour periods. [1]
    b. Find the population after 3 days. [2]
    c. After how many 8-hour periods will the population exceed **1,000,000 cells**? [2]

Answer Key — 1.3.1 Geometric Sequences (Part 1)

Practice Set A — Routine Practice

1) $a_1=7$, $r=4$. Find a_6.

Formula: $a_n = a_1 r^{n-1}$
Substitute: $a_6 = 7 \cdot 4^5$
Simplify: $4^5 = 1024 \Rightarrow a_6 = 7 \cdot 1024 = 7168$
Answer: 7168

2) $a_1=1250$, $r=0.8$. Find a_5.

Formula: $a_n = a_1 r^{n-1}$
Substitute: $a_5 = 1250 \cdot (0.8)^4$
Simplify: $(0.8)^4 = 0.4096 \Rightarrow a_5 = 1250 \cdot 0.4096 = 512$
Answer: 512

3) $a_3=81$, $r=3$. Find a_1.

Formula: $a_n = a_1 r^{n-1}$
Substitute: $81 = a_1 \cdot 3^2$
Solve: $a_1 = 81/9 = 9$
Answer: 9

4) Sum of first 5 terms, $a_1=2$, $r=5$.

Formula: $S_n = a_1(1 - r^n) / (1 - r)$
Substitute: $S_5 = 2(1 - 5^5)/(1 - 5)$
Simplify: $2(1 - 3125)/(-4) = 1562$
Answer: 1562

5) $a_1=640$, $r=1/2$. Find S_6.

Formula: $S_n = a_1(1 - r^n)/(1 - r)$
Substitute: $S_6 = 640(1 - (1/2)^6)/(1 - 1/2)$
Simplify: $(1/2)^6=1/64 \Rightarrow S_6=640(63/64) \cdot 2=1260$
Answer: 1260

6) $a_4=54$, $r=3$. Find S_4.

Find a_1: $a_4 = a_1 \cdot r^3 \Rightarrow a_1 = 54/27 = 2$
Formula: $S_n = a_1(1 - r^n)/(1 - r)$
Substitute: $S_4 = 2(1 - 3^4)/(1 - 3) = 80$
Answer: 80

Practice Set B — IB-Style (Sequences Only)

Leo's Lawn Business

a(i) Day 2 price = 40 + 5 = 45.

a(ii) Price on day n: $p_n = 40 + 5(n - 1) = 35 + 5n$.

a(iii) Lawns on day n: $\ell_n = 2 + (n - 1) = n + 1$.

b) $35 + 5n > 100 \Rightarrow n > 13 \Rightarrow n = 14$.

c) $\ell_n = 10 \Rightarrow n + 1 = 10 \Rightarrow n = 9$.

2) Ava's Subscription vs Bea's Subscription

a(i) Year 1 = \$10, Year 2 = \$12.

a(ii) $c_n = 10 + 2(n - 1) = 2n + 8$.

a(iii) $c_4 = 2(4) + 8 = \$16$.

b(i) $b_n = 10(1.05)^{n-1}$.

b(ii) $b_3 = 10(1.05)^2 \approx \$11.03$.

b(iii) Year 4: Ava = \$16, Bea = $10(1.05)^3 \approx \$11.58 \Rightarrow$ Ava pays more.

3) Cycling Distance

$d_n = 12(1.15)^{n-1}$.

$12(1.15)^{n-1} > 50 \Rightarrow n \approx 12$.

$d_7 = 12(1.15)^6 \approx 27.8$ km.

4) Coffee Price

$p_n = 3.20(1.10)^{n-1}$.

$p_5 = 3.20(1.10)^4 \approx \$4.69$.

$3.20(1.10)^{n-1} > 5 \Rightarrow n \approx 6$.

5) Bacterial Culture

$P_n = 1200 \cdot 3^n$.

$P_9 = 1200 \cdot 3^9 \approx 23{,}619{,}600$.

$1200 \cdot 3^n > 1{,}000{,}000 \Rightarrow n \approx 7$.